

Gravitation at the mesoscopic scale *

R. Onofrio^{1*} and L. Viola^{2†}

¹ *Dipartimento di Fisica “G. Galilei”, Università di Padova, and INFN, Sezione di Padova,
Via Marzolo 8, Padova 35131, Italy*

² *Department of Physics, Massachusetts Institute of Technology, 12-127,
77 Massachusetts Avenue, Cambridge, MA 02139, USA*

(February 1, 2008)

Free fall experiments are discussed by using test masses associated to quantum states not necessarily possessing a classical counterpart. The times of flight of the Galileian experiments using classical test masses are replaced in the quantum case by probability distributions which, although still not defined in an uncontroversial manner, become manifestly dependent upon the mass and the initial state. Such a dependence is also expected in non inertial frames of reference if the weak equivalence principle still holds. This last could be tested, merging recent achievements in mesoscopic physics, by using cooled atoms in free fall and accelerated frames initially prepared in nonclassical quantum states.

KEY WORDS: Gravitation, Quantum Mechanics

Quantum mechanics and general relativity are still the milestones of modern physics in the closing part of this century. Their fields of applicability were not originally overlapping, since quantum mechanics was developed for microscopic phenomena where gravitational effects were expected to play a negligible role and, vice versa, gravity manifests in the macroscopic world where no quantum phenomena were evidenced. The situation has been changing in more recent times due both to possible astrophysical observations and controllable laboratory experiments. Pioneered by Hawking, quantum phenomena were predicted for highly massive relativistic objects such as black holes [1]. On the other hand, thanks to the astonishing development of new experimental techniques, the frontier of applicability of quantum mechanics itself has been pushed further in the macroscopic realm by crossing an intermediate *mesoscopic* region where, albeit no more on a properly microscopic scale, quantum features still dominate. These developments, although motivated by the need to understand more neatly the macroscopic-microscopic borderline in quantum phenomena, affect gravitation as well. Since the current belief is that quantum laws are the fundamental ones, the fact that gravitational laws have been formulated in terms of macroscopic test bodies is accidental. Hence it should be relevant to discuss how to describe gravitation operationally without mention of classical concepts. This is also motivated by many attempts and proposals to investigate gravitation using microscopic and mesoscopic systems, such as antimatter in free fall [2,3], superfluid and superconducting systems [4], cooled atoms in optical molasses [5] and opto-gravitational cavities [6], interfering matter waves both in curved space-time and accelerated reference frames [7–10], neutrinos acquiring phases of gravitational origin [11].

Our essay deals with the issue of establishing the foundational basis of gravitation by using quantum objects not necessarily possessing a classical counterpart [12–14]. It turns out that, together to sharpen some well-established hypotheses and to show the emergence of some conceptual problems, the way is open to test gravitation in a Lilliput world made by atoms prepared in mesoscopic states. Among various aspects, we will focus on the weak equivalence principle. Galileo discussed the universality of the ratio between gravitational and inertial masses by imagining test bodies in free fall from the tower of Pisa [15]; since then several actual tests have been performed with very sensitive schemes [16], but only involving classical test bodies. It is, therefore, natural to revisit the Pisa Gedankenexperiment by using properly prepared quantum objects.

Given two particles with inertial masses $m_i^{(1)}$, $m_i^{(2)}$, $m_i^{(1)} \neq m_i^{(2)}$, we want to prepare them in such a way that any possible difference during the free fall motion must be ascribed to the effect of gravity. This is more subtle than expected due to the Heisenberg uncertainty principle [13,17]. By denoting with $|\psi_1\rangle$, $|\psi_2\rangle$ the initial state for particles 1, 2 in the Schrödinger picture, we rephrase the Galileian prescription for initial positions and velocities by constraining the expectation values of the position and momentum operators in the following way:

$$\langle \hat{z} \rangle_{\psi_1} = \langle \hat{z} \rangle_{\psi_2}, \quad \frac{\langle \hat{p}_z \rangle_{\psi_1}}{m_i^{(1)}} = \frac{\langle \hat{p}_z \rangle_{\psi_2}}{m_i^{(2)}}, \quad (1)$$

*This essay received “honorable mention” from the Gravity Research Foundation, 1997 — Ed.

*E-mail: Onofrio@padova.infn.it

†E-mail: Vlorenza@mit.edu

where for simplicity we limit the attention to a one-dimensional nonrelativistic motion along the z -direction. Eq. (1) implies, in general, that two different initial states $|\psi_1\rangle \neq |\psi_2\rangle$ in the single particle Hilbert space have to be selected: this is analogous to the classical situation, where the representative point of the classical initial state in the phase space is different for the two masses. On the other hand, unlike the classical case, the probabilistic interpretation underlying quantum mechanics only allows to speak of *mean* initial conditions like (1), which are far from univocally determining the initial state of the two particles: rather, they give rise to equivalence classes of states, each one characterized by an average position and velocity and collapsing in the classical limit into a state with both quantities sharply defined. Strictly speaking, this holds whenever quantum states admitting classical analogue are considered. Earlier remarks by Einstein and Schrödinger showed that such a class of states by no means exhausts the totality of the admissible ones in the Hilbert space. Indeed, due to the superposition principle, states of intrinsically quantum nature arise even starting from states which in the classical limit correspond to macroscopically distinguishable ones. If $|\psi_n\rangle$, $n = 1, \dots, N$, denote a set of macroscopically distinct states of a given quantum system, any superposition $|\psi_0\rangle = \sum_n c_n |\psi_n\rangle$ is also permitted, with the complex coefficients c_n ensuring normalization. The nonclassical nature of this superposition state is manifest from its density matrix representation,

$$\hat{\rho} = |\psi_0\rangle\langle\psi_0| = \sum_n |c_n|^2 |\psi_n\rangle\langle\psi_n| + \sum_{n,m \neq n} c_n^* c_m |\psi_n\rangle\langle\psi_m|, \quad (2)$$

the off-diagonal terms being responsible for correlations of a purely quantum mechanical origin. In the classical limit, due to the action of decoherence mechanisms, interference effects are lost, and the pure state description (2) becomes identical to the statistical mixture characterized by the diagonal probability weights $|c_n|^2$ alone. For a generic observable its overall mean value in the state $|\psi_0\rangle$ is thought as formed, according to (2), by two distinct terms, one surviving in the classical limit and another one made by the purely quantum average over non-diagonal matrix elements. A nice class of quantum states without classical counterpart is offered by the so-called Schrödinger cat states [18], whose simplest example is the coherent superposition of two macroscopically distinguishable states in the configurational space, with wavefunction

$$\psi_0(z) = \mathcal{N} \left\{ c_+ \exp \left(- \frac{(z - z_0 + \Delta)^2}{2\Delta_0^2} \right) + c_- \exp \left(- \frac{(z - z_0 - \Delta)^2}{2\Delta_0^2} \right) \right\}, \quad (3)$$

consisting of a properly normalized sum of two Gaussian peaks of width Δ_0 at $z = z_0 \pm \Delta$, $\Delta > 0$. In case $\Delta = 0$ a standard Gaussian wavepacket is recovered. By denoting with ϑ the relative phase between the complex coefficients c_{\pm} , the expectation values of position and momentum in the state (3) are calculated obtaining:

$$\langle \hat{z} \rangle_{\psi_0} = z_0 - \frac{\Delta}{\Delta_0} \frac{|c_+|^2 - |c_-|^2}{\sqrt{\pi} |\mathcal{N}|^2}, \quad \langle \hat{p}_z \rangle_{\psi_0} = - \frac{2\hbar}{\Delta_0^2} \frac{\Delta}{\Delta_0} \exp(-\Delta^2/\Delta_0^2) \frac{|c_+||c_-| \sin \vartheta}{\sqrt{\pi} |\mathcal{N}|^2}. \quad (4)$$

By Fourier transforming (3), it is seen that no diagonal momentum contributions are present, leading to a purely quantum momentum in (4) with the form of a typical interference factor. A vanishing value of $\langle \hat{p}_z \rangle_{\psi_0}$ is found if the relative phase $\vartheta = k\pi$, $k \in \mathbb{Z}$, corresponding to the even (male, $c_+ = c_- = 1$) and odd (female, $c_+ = -c_- = 1$) combinations of definite parity cat states [19]; a maximum contribution is instead achieved at $\vartheta = (2k + 1)\pi/2$, $k \in \mathbb{Z}$, corresponding to cat wavefunctions already introduced by Yurke and Stoler [20]. For the choice of two cat-like quantum particles of mass $m_i^{(1)}, m_i^{(2)}$, it is amazing to realize that prescriptions (1), although no more supported by any correspondence arguments, can still be fulfilled by suitably gauging the parameters of the wavefunction (3). Having checked this possibility in general, we will hereafter shorten formulas by assuming $\langle \hat{p}_z \rangle_{\psi_0} = 0$.

Once the initial preparation of each particle is specified, let us consider the simplest case of a time evolution generated by the Hamiltonian $\hat{H} = \hat{p}_z^2/2m_i + m_g g \hat{z}$, m_g denoting the gravitational mass, and let us focus, in analogy with Galileo's procedure, on the times of flight of the quantum particles from the initial height z_0 to a ground reference level. Even in the absence of any statistical uncertainty, one must then face the fact that time of flight probability distributions, instead of well-defined values as in the classical case, are in principle demanded to gain complete information on the free fall dynamics. It is perhaps surprising that no general consensus has been reached so far on a consistent definition of time of flight probability densities, even if various attempts have been made until recent times [21]. Leaving aside a rigorous derivation, we can limit ourselves to semiclassical arguments. One can then infer the average time of flight $T_{of}^{(k)}$ for the test mass $m_i^{(k)}$ by inverting the relation for the average position of the particle, which is available from Ehrenfest's theorem:

$$\langle z^{(k)}(t = T_{of}^{(k)}) \rangle = z_0 - \frac{1}{2} \frac{m_g^{(k)}}{m_i^{(k)}} g t^2 = 0, \quad k = 1, 2. \quad (5)$$

A rough estimate of the fluctuations around its mean value, taking into account the spreading of the state during the motion, can be given by $\sigma_{T_{of}} \approx \sigma_z(T_{of})/v_z(T_{of})$, $\sigma_z^2(T_{of})$ and $v_z(T_{of})$ being the position variance and the average velocity at time $t = T_{of}$ respectively. The general expression for $\sigma_z^2(t) = \langle \hat{z}^2 \rangle_{\psi_t} - \langle \hat{z} \rangle_{\psi_t}^2$ can be evaluated [13]. In an intermediate regime where $\Delta \approx \Delta_0$ and nonclassical effects are maximally enhanced, the result can be cast in a simple form so that:

$$T_{of}^{(k)} \pm \sigma_{T_{of}}^{(k)} = \sqrt{2 \left(\frac{m_i^{(k)}}{m_g^{(k)}} \right) \frac{z_0}{g}} \pm \frac{\sqrt{2}}{2} \epsilon \frac{\hbar}{\Delta_0 m_g^{(k)} g}, \quad k = 1, 2, \quad (6)$$

being ϵ a numerical factor equal to 1 for a Gaussian state and to $[(e-1)/(e+1)]^{\pm 1/2}$ for a male (+) or a female (−) cat state. It is manifest from (6) that, despite the semiclassical preparation recipe (1), the time of flight distributions depends upon the different choices of masses and/or initial states. The ratio between inertial and gravitational mass contributes to the average time of flight, whereas the fluctuations around this value are affected by the *gravitational* mass alone. Once any state difference is removed by choosing initial states of the same kind, a mass dependence still survives even if, in close analogy with the classical experiment, the equality $m_i^{(1)}/m_g^{(1)} = m_i^{(2)}/m_g^{(2)}$ is established from observing identical values of the average time of flight. In other words, the widespread quoted universal behavior in a gravitational field breaks down when quantum test bodies are included. However, this does not necessarily prevent quantum effects from being universally influenced by the interaction with gravity, provided one can interpret *any* dependence on the mass parameter within a purely *kinematical* framework. In our problem, formally identical equations indeed hold for the motion of a quantum particle freely evolving in a non-inertial reference frame accelerating with $a = g$, and identical probability distributions are, therefore, predicted for similar time of flight experiments [13]. In general, universal classical behavior is replaced by mass dependent quantum observables, but such a dependence is expected - with an identical structure for motions occurring in a gravitational or an accelerated laboratory - in order Einstein's equivalence principle to be preserved at the quantum level. Note that this specific feature can be deeply related to the impossibility of reproducing, for any quantum object, the classical concept of a deterministic trajectory. In the Nelson's picture of quantum mechanics [22], where the kinematics is modeled through stochastic configurational trajectories, it is not surprising that the combination \hbar/m ultimately appears in (6), *i.e.* the Brownian diffusion coefficient accounting for the degree of stochasticity. This inevitable randomness, which preexists to the introduction of the gravitational field itself, is a source of troubles in extending the geometric interpretation which so beautifully underlies gravity at the classical level. Indeed, the possibility of a simple identification between the world lines of freely falling bodies and a set of preferred entities with a purely geometric nature clearly no more holds [23–25]. At the same time, this difficulty has been also pointed out as the motivation to introduce a new definition of the equivalence principle [26]. Moreover, it has been recently evidenced that non-geometric features of gravity appear by considering quantum mechanical clocks based on flavor-oscillations [14].

From the experimental viewpoint, no evidence exists to date that the description given above actually fits the significance of the equivalence principle in the quantum realm. Nevertheless, the impressive progress attained in the manipulation of atomic states gives hope to make experimental investigation feasible. We only recall here that time of flight measurements on the free fall of a Gaussian cloud of cooled Cesium atoms were already performed in a semiclassical regime [6,27] and, on the other hand, Schrödinger cats of a single trapped Be^+ ion have been also generated in laboratory [28]. Merging these extraordinary accomplishments, a mesoscopic experimental test of the equivalence principle can be envisaged, whereby a Gaussian or a Schrödinger cat state of matter at the single atom level is subjected to a gravitational or an accelerating field.

The relevance of mesoscopic physics for exploring gravitation is not limited to a proper establishment of the equivalence principle. First, deviations from the universal Newton law due to microscopic models of gravity have been proposed [29] and can be investigated at a submillimeter lengthscale with micromechanical devices [30]. In addition, after the recent spectacular achievement of Bose-Einstein condensation [31], a new ultracold state of matter is available and can be used to probe the tiny energy scales associated to gravitationally bound states. As a general remark, mesoscopic dynamics is extremely sensitive to perturbations introduced by the surrounding environment. Therefore, the effect of any interrogation on the system should be taken into account, ultimately implying that any operative statement about gravitational properties has to be given consistently with quantum measurement theory [32,33]. As emphasized in [13], the search for a unified picture in this framework will not unlikely require the emergence of new concepts in gravitation.

Although still in its infancy, mesoscopic gravitation gives promising directions to investigate gravity in a landscape where it is forced to convive with quantum mechanical laws.

ACKNOWLEDGMENTS

We are grateful to Clair Mason for a critical reading of the manuscript.

-
- [1] S. W. Hawking, *Commun. Math. Phys.* **43**, 199 (1975).
 - [2] F. C. Witterborn, and W. M. Fairbank, *Phys. Rev. Lett.* **19**, 1049 (1967); W. M. Fairbank, F. C. Witterborn, J. M. J. Madey, and J. M. Lockhart, in *Experimental Gravitation*, edited by B. Bertotti (Academic Press, New York, 1989), p. 310.
 - [3] T. Goldman, R. J. Hughes and M. M. Nieto, *Phys. Lett. B* **171**, 217 (1986); M. N. Nieto and T. Goldman, *Phys. Rep.* **205**, 5 (1991); V. Lagomarsino, V. Lia, G. Manuzio, and G. Testera, *Phys. Rev. A* **50**, 977 (1994).
 - [4] J. Anandan, in *New Techniques and Ideas in Quantum Measurement Theory*, edited by D. M. Greenberger, *Annals of the New York Academy of Sciences*, vol. 480 (New York Academy of Sciences, 1986), p. 224.
 - [5] M. A. Kasevich, E. Riis, S. Chu, R. G. DeVoe, *Phys. Rev. Lett.* **63**, 612 (1989).
 - [6] C. G. Aminoff, A. M. Steane, P. Bouyer, P. Desbiolles, J. Dalibard, and C. Cohen-Tannoudji, *Phys. Rev. Lett.* **71**, 3083 (1993).
 - [7] R. Colella, A. W. Overhauser, and S. Werner, *Phys. Rev. Lett.* **34**, 1472 (1975).
 - [8] B. Mashhoon, *Phys. Rev. Lett.* **61**, 2639 (1988).
 - [9] M. Kasevich and S. Chu, *Phys. Rev. Lett.* **67**, 181 (1991).
 - [10] J. Audretsch and R. Müller, *Phys. Rev. A* **50**, 1755 (1994); **52**, 629 (1995).
 - [11] D. V. Ahluwalia and C. Burgard, *Gen. Rel. Grav.* **28**, 1161 (1996).
 - [12] R. Penrose, in *Three hundreds years of gravitation*, edited by S. W. Hawking and W. Israel (Cambridge University Press, Cambridge, 1987), p. 31.
 - [13] L. Viola and R. Onofrio, *Phys. Rev. D* **55**, 455 (1997).
 - [14] D. V. Ahluwalia, *On a New Non-Geometric Element in Gravity*, Report No. gr-qc/9705050, to appear in *Gen. Rel. Grav.* (1997).
 - [15] G. Galilei, *Discorsi intorno a due nuove scienze* (Leiden, 1638) [English translation edited by H. Crew and A. de Salvio (Macmillan, New York, 1914), pp. 212-213].
 - [16] C. M. McWill, *Theory and Experiments in Gravitational Physics* (Cambridge University Press, Cambridge, 1991); *Experimental Gravitation*, edited by M. Karim and A. Qadir (Institute of Physics Publishing, Bristol and Philadelphia, 1994).
 - [17] S. T. Epstein, *Phys. Lett. A* **11**, 233 (1964).
 - [18] E. Schrödinger, *Naturwissenschaften* **23**, 812 (1935); Letter from Einstein to Schrödinger of 22 December 1950, in *Briefe zur Wellenmechanik*, edited by K. Przibram (Springer, Vienna, 1963), p. 36.
 - [19] V. V. Dodonov, I. A. Malkin and V. I. Man'ko, *Physica* **72**, 597 (1974).
 - [20] B. Yurke and D. Stoler, *Phys. Rev. Lett.* **57**, 13 (1986).
 - [21] N. Grot, C. Rovelli and R. S. Tate, *Phys. Rev. A* **54**, 4676 (1996), and references therein.
 - [22] E. Nelson, *Phys. Rev.* **150**, 1079 (1966); *Quantum Fluctuations* (Princeton University Press, Princeton, NJ, 1985).
 - [23] S. Sonego, *Phys. Lett. A* **208**, 1 (1995).
 - [24] Y. Aharonov and T. Kaufherr, *Phys. Rev. D* **30**, 368 (1984); C. Rovelli, *Class. Quantum Grav.* **8**, 297 (1991); 317 (1991); M. Toller, *Quantum Reference Frames and Quantum Transformations*, Report No. gr-qc/9605052.
 - [25] L. M. Morato and L. Viola, *J. Math. Phys.* **36**, 4691 (1995).
 - [26] C. Lämmerzahl, *Gen. Rel. Grav.* **28**, 1043 (1996).
 - [27] A. Steane, P. Szriftgiser, P. Desbiolles, and J. Dalibard, *Phys. Rev. Lett.* **74**, 4972 (1995).
 - [28] C. Monroe, D. M. Meekhof, B. E. King, and D. J. Wineland, *Science* **272**, 1131 (1996).
 - [29] M. Gasperini, *Gen. Rel. Grav.* **16**, 1031 (1984); T. Damour and A. M. Polyakov, *Nucl. Phys. B* **423**, 532 (1994); *Gen. Rel. Grav.* **20**, 1171 (1994); D. Dimopoulos and G. F. Giudice, *Phys. Lett. B* **379**, 105 (1996); R. Barbieri, A. Romanino and A. Strumia, *Phys. Lett. B* **387**, 310 (1996).
 - [30] G. Carugno, Z. Fontana, R. Onofrio, and C. Rizzo, *Limits on the existence of scalar interactions in the submillimeter range*, to appear in the May 15th issue of *Phys. Rev. D* (1997).
 - [31] M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell, *Science* **269**, 198 (1995); K. B. Davis, M.-O. Mewes, M. R. Andrews, N. J. van Druten, D. S. Durfee, D. M. Kurn, and W. Ketterle, *Phys. Rev. Lett.* **75**, 3969 (1995).
 - [32] C. Presilla, R. Onofrio and U. Tambini, *Ann. Phys. (NY)* **248**, 95 (1996).
 - [33] C. Lämmerzahl and C. J. Bordé, *Phys. Lett. A* **203**, 59 (1995); I. Percival, *Atomic Interferometry, spacetime and reality*, *Physics World*, March 1997, p. 43.